Low Phase Noise Oscillators: Theory and Application

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***Abstract*— This paper reviews a linear model and theory for the design of low phase noise oscillators. A description of how an oscillator works and the important parameters are then described in detail. A set of design rules are developed for fixed frequency and tunable oscillators. A number of resonators which use the theory to produce oscillators which demonstrate phase noise performance usually within 0 to 1dB of the theoretical minimum are described. Some of the oscillators demonstrate the best phase noise performance available.**

I. Introduction

The oscillator in communication and measurement systems, defines the reference signal onto which modulation is coded and later demodulated. The flicker and phase noise in such oscillators are central in setting the ultimate systems performance limits of modern communications, radar and timing systems. These oscillators are therefore required to be of the highest quality for the particular application as they provide the reference for data modulation and demodulation.

This paper describes a model and linear theory for low phase noise oscillators. This theory is used to develop a set of design rules which typically enable oscillators to be designed with performance within ~0 to 1dB of the theoretical minimum performance.

II. PHASE NOISE THEORY

It is important to develop a simple model to calculate and predict the noise performance of an oscillator. Leeson [1] demonstrated an equation which gives useful information about the phase noise but the optimum conditions for minimum noise are not clear. Parker [2] demonstrated an optimum condition for a modified version of Leeson's equation. These models were developed by analyzing the baseband modulation process for an oscillator. It is however useful, to develop a simple model, from first principles, which enables an accurate and clearly understood equation to be derived.

A suitable model is shown in figure 1 [3][4]. This consists of an amplifier with two inputs which are added together. These represent the same input but are separated to enable one to be used to model the noise input and the other for feedback. The resonator is represented as an LCR circuit where any impedance transformation is achieved by varying the component values.

It is then possible to calculate the voltage transfer function (at the operating frequency) and then the noise power transfer function. This circuit, through positive feedback, operates as a Q multiplication filter but also contains the additional constraint that the AM noise is suppressed in the limiting process. This causes the upper and lower sidebands to become coherent and this has been defined as conformability by Robbins [5]. The model is put in this form to highlight all the effects, which are often not clear in a block diagram model.

A general equation for the single sideband phase noise can be derived as shown in equation 1 [3] which allows for a number of operating conditions including power and the output and input impedances. F is the operating noise factor which includes the amplifier parameters under the oscillating operating conditions, k is the Boltzmann constant and T is the operating temperature. Q_0 is the unloaded Q, Q_L is the loaded Q, f_0 is the oscillator frequency and f is the offset frequency. P is the power in the oscillator which can be defined in two ways. P_{AVO} is the power available from the output of the amplifier and P_{RF} is the total power dissipated in the input, output, and equivalent loss resistances. N and A are integer variables which could be 1 or 2 dependent on the definition of P.

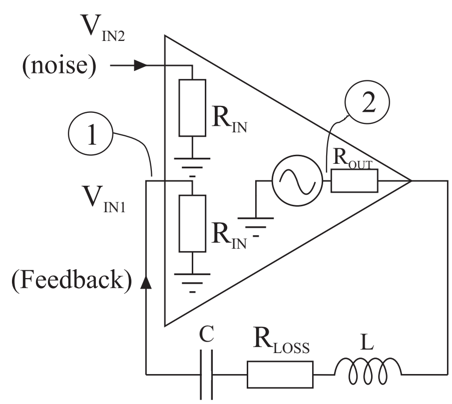

Figure 1 Oscillator Model

$$L(f) = A.\frac{FkT}{8(Q_0)^2(Q_L/Q_0)^2(1-Q_L/Q_0)^N P}\left(\frac{f_0}{f}\right)^2 \quad (1)$$

where:

1. $N = 1$ and $A = 1$ if P is defined as P_{RF} and R_{OUT} = zero.
2. $N = 1$ and $A = 2$ if P is defined as P_{RF} and $R_{OUT} = R_{IN}$.
3. $N = 2$ and $A = 1$ if P is defined as P_{AVO} and $R_{OUT} = R_{IN}$.

If we take expansion 3 and show the full equation including the impedances, we obtain equation 2 [3].

$$L(f) = \frac{FkT}{32(Q_0)^2(Q_L/Q_0)^2(1-Q_L/Q_0)^2 P_{AVO}}\left(\frac{(R_{OUT}+R_{IN})}{R_{OUT}.R_{IN}}\right)\left(\frac{f_0}{f}\right)^2 \quad (2)$$

The bracket including R_{OUT} and R_{IN} is minimum when $R_{OUT} = R_{IN}$ causing the whole bracket to equal 4. Equation 2 then simplifies to equation 3:

$$L(f) = \frac{FkT}{8Q_0^2\left(\frac{Q_L}{Q_0}\right)^2\left(1-\frac{Q_L}{Q_0}\right)^2 P_{avo}}\left(\frac{f_0}{f}\right)^2 \quad (3)$$

This equation will be used in the analysis for the noise performance and gain requirements.

A plot of the phase noise variation vs the parameters is shown in Figure 2.

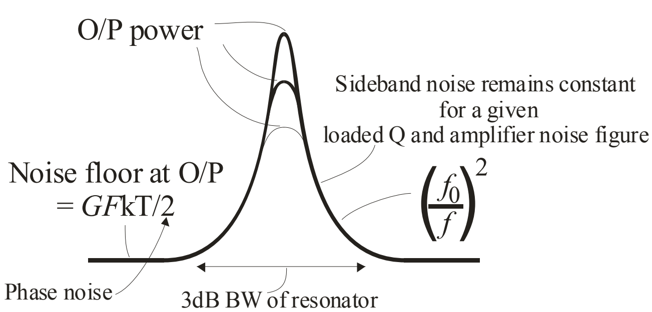

Figure 2 Phase Noise Variation with parameters

It is interesting to make some comments about the phase noise based on this model and these equations.

- The circuit operates as a Q multiplier.
- The 3dB point of the resonator becomes the 3dB point ABOVE the noise floor.
- In the thermal noise regime (excluding flicker noise) the Q is multiplied by approximately the ratio of the output voltage to the input voltage.
- This Q multiplication is huge.
- This model implies that $\beta_0 G$ approaches 1 but is always slightly less than one. It becomes closer to 1 as the power is increased.
- The phase noise sidebands remain constant for a given loaded Q and amplifier noise figure.
- The ratio of single sideband noise to total output power (L(f) therefore increases linearly with power
- The noise sidebands roll off at $(1/f)^2$
- A doubling of the carrier frequency causes 6dB (f_0^2) degradation in phase noise.
- The phase noise improves with Q_0^2.

Equation 3 has a further minimum when $Q_L/Q_0 = ½$ and hence when the insertion loss of the resonator is 0.25 (–6dB). This was first described by Parker in [2]. This minimum occurs because the amplifier gain is set by the insertion loss of the resonator which is:

$$S_{21} = (1 = Q_L/Q_0) \quad (4)$$

Under optimum conditions:

$$L(f) = \frac{2FkT}{Q_0^2 P_{AVO}}\left(\frac{f_0}{f}\right)^2 \quad (5)$$

Equation (5) is similar to Leeson's equation [1], although Leeson's equation used the loaded Q, Q_L, and the power incident on the input, P_I. Taking Leeson's Equation from [1] where the flicker noise parameters and the noise floor are left out, we obtain:

$$S_\phi(f) = \frac{FkT}{2Q_L^2 P_I}\left(\frac{f_0}{f}\right)^2 \quad (6)$$

where $S_\phi(f)$ is the spectral power density of phase fluctuations in radians squared per Hertz.

Taking the conditions that $P_I = P_{AVO}/4$ and $Q_L = Q_0/2$, based on the optima derived from equation (3), and assuming small angle modulation, then Leeson's equation becomes:

$$L(f) = \frac{S_\phi(f)}{2} = \frac{4FkT}{Q_0^2 P_{AVO}}\left(\frac{f_0}{f}\right)^2 \quad (7)$$

This version of Leeson's equation has the same form as equation (3) but is twice it's value. This is because equation (3) incorporates the assumption that limiting causes the phase noise component to be half the total thermal noise.

This lower noise floor is confirmed in [6] where they have shown ‘theoretically and experimentally that the single sideband PM (and AM) noise floor due to thermal noise’ (at room temperature) ‘is -177dBc/Hz relative to a carrier input power of 0dBm’.

It should be noted that the noise factor, F, was assumed to be constant in this optimization. The noise factor may however vary with source impedance and hence Q_L/Q_0 as described in [3]. The noise factor may also show a power dependence which can be affected by both the device type and the amplifier topology. Examples of the effects of gain compression on the thermal (additive noise) and transposed flicker noise (modulation noise) are shown in [7]. If the inter-dependence is known then this can be incorporated to modify the optimum value of Q_L/Q_0.

To obtain further insight, a plot of noise degradation with resonator insertion loss and hence closed loop gain is shown in Figure 4 (based on equation 1, assumption 3). This gives a more obvious indication of the allowable tolerances on the insertion loss of a resonator. It can be seen that only one dB of noise degradation occurs when the insertion loss is within the bounds of 3.5 to 9.5dB.

The requirements for minimum noise are therefore:

1. Q_0 is as high as possible
2. P_{AVO} is maximum and noise figure is minimum
3. $Q_L/Q_0 = ½$, therefore S_{21} of the resonator should be set to -6dB

It should be noted that the optima, just discussed, apply if the noise is thermal (additive) noise and also only apply to the skirts of the phase noise. For far out noise to be minimum the gain should be kept low (Q_L/Q_0 low) and for reduced transposed flicker noise the loaded Q should be higher. However this optimum is a good starting point.

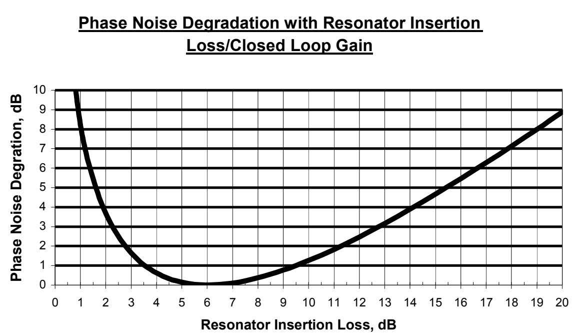

Figure 4, Phase noise degradation with resonator insertion loss/open loop gain

Equation 3 can be extended to describe the phase noise including the effect of the noise floor, the transposed flicker noise and the noise contributions of the output couple and buffer amplifier. This is shown below in equation 8. The right hand term is based on equation 3, the middle term shows noise outside the resonator bandwidth (far from carrier noise) and both these terms are multiplied by a flicker noise component ($1 + F_C/f$). The left hand term includes the buffer amplifier after the output coupler and is still assumed to be limited by the phase noise measurement (therefore 2P). The ‘1’ refers to the phase noise of a single oscillator. This is changed to 2 when the combined noise of two identical oscillators is being displayed.

$$L(f)=10\mathrm{Log}\left[\left[1\left[\frac{F_2 kT}{C_0 2P}+\left(1+\frac{Fc}{f}\right)\left(\frac{F_1 kT}{2P}\left(\frac{1}{\left(1-\frac{Q_L}{Q_0}\right)}\right)+\frac{F_1 kT}{8(Q_0)^2\left(\frac{Q_L}{Q_0}\right)^2\left(1-\frac{Q_L}{Q_0}\right)^2 P}\left(\frac{f_0}{f}\right)^2\right)\right]\right]\right] \quad (8)$$

III. OSCILLATOR CONFIGURATION

A typical block diagram of a feedback oscillator is illustrated in Figure 5. This consists of an amplifier, output coupler, filter (if necessary) to ensure operation at the correct resonant frequency, resonator (fixed frequency of tunable), and if required a voltage controlled phase shifter for narrow band tuning and fixed phase shifter to ensure Nx360 degree phase shift at the centre of the resonant frequency.

It should also be noted that for accurate analysis and noise prediction, all the items other than the resonator should be combined into an ‘equivalent amplifier block’ and the required Noise Figure, open loop gain and P_{AVS} should be calculated and/or measured. We have found, using these techniques, that the noise predictions are almost always within 0 to 1dB of the theory.

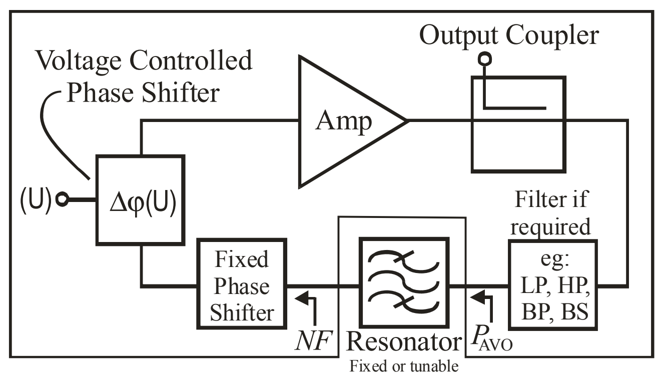

Figure 5, Generic Block Diagram for a feedback oscillator. Assume all elements inside the box are part of the amplifier for calculation of phase noise using Noise Factor (*NF*) and P_{AVO}

To achieve best performance each element should be designed and optimized separately. The major parameters for the loop amplifiers, resonators and tuning elements will now be described.

A. AMPLIFIERS

The amplifiers should offer:

1. Well defined input and output impedance

2. Low noise figure
3. Low Residual Flicker Noise using optimum bias. Flicker noise corners < 5kHz should be achievable at RF and microwave frequencies and < 20Hz at low frequencies
4. Low dependence on power supply noise and low regulator noise
5. Reasonable level of open loop gain ~ at typical value around 10dB is often reasonable
6. Well defined limited output power
7. Smooth monotonic saturation characteristics with negligible change in parameters
8. Unconditional stability both in the linear and saturation regions

Many of these parameters are interdependent and therefore need to be optimized carefully.

B. Resonators

Resonators should offer:

1. High unloaded Q. It is important to ensure uniform current distribution in metal surfaces and this is usually improved with symmetry.

2. Low and where possible far removed spurious responses. If required additional filters can be incorporated to ensure sufficient loss at the unwanted resonance.

3. Correct coupling: Design the coupling network to achieve an insertion loss of 6dB. Note that only 1dB noise degradation occurs if the insertion loss is within the bounds of -3.5 to -9.5dB. This is also the same for tunable resonators where the losses in the varactor are incorporated into the Q calculations.

4. Low sensitivity to external parameters such as temperature and vibration

C. TUNING ELEMENTS

1) Phase Shifter Method

Frequency tuning is typically achieved by incorporating the varactor diodes either into the resonator or by using phase shift tuning separate from the resonator as illustrated in Figure 4. Phase shift tuning has the advantage that it does not degrade the resonator unloaded Q and the phase noise degradation can be accurately calculated. This group has shown (theoretically and experimentally) that the noise performance degrades with a $\cos^4\theta$ relationship [3] [7]. Therefore an open loop phase error of 45º causes 6dB degradation + the insertion loss of the phase shifter.

The phase shifter should have low insertion loss and a near linear phase vs frequency response. It is important to ensure that the change in insertion loss and phase shift does not change significantly with power level. In fact this is even more critical in varactor tuned resonators where the Q is higher.

2) Varactor controlled resonator method

The noise performance of a broad tuning range oscillator is usually limited by the Q and the voltage handling capability of the varactor as has been described by Underhill [8]. These equations were extended to include oscillators operating under optimum conditions [3], [9], and are described here for general conditions which relate this operating AC voltage to the coupling coefficient and hence Q_L/Q_0. This is examined by taking a simple model for the resonator as shown in Figure 6. This consists of a series LCR network which can be used to model both series and parallel resonators. All the losses from the varactor, inductor and capacitor are combined into R_{LOSS}.

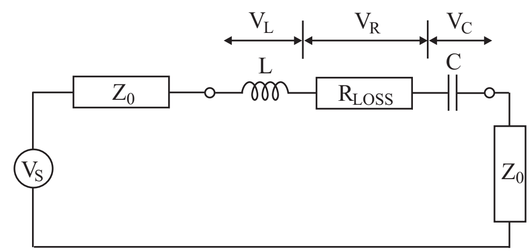

Fig. 6. General Model for Resonator

$$P_{RLOSS} = 2\frac{Q_L}{Q_0}\cdot\left(1-\frac{Q_L}{Q_0}\right)\cdot P_{AVO} \tag{17}$$

$$V_C = Q_L V_S$$

When $Q_L/Q_0 = 1/2$, $P_{RLOSS} = P_{AVO}/2$. If it is assumed that the varactor losses are dominant then the noise performance is set totally by the varactor loss resistance and the voltage handling capability to be [3] [8]:

$$L(f) = \frac{FkTR_{LOSS}}{V_C^{\,2}}\left(\frac{f_0}{\Delta f}\right)^2 \tag{18}$$

Note that if a parallel resonator is used then this series model can still be used by converting the parallel loss resistances R_P to series loss resistances R_S. For reasonable values of Q where R_S would then be $(\omega L)^2/R_P$:

It is interesting to note that non linear effects occur at surprisingly low power levels as illustrated in Figure 7. This consists of an LC resonator with a loaded Q of 63 with 3 volts DC bias on the varactors. Distortion starts to occur at -6dBm! This onset of distortion occurs when the varactor AC voltage is around the DC bias voltage.

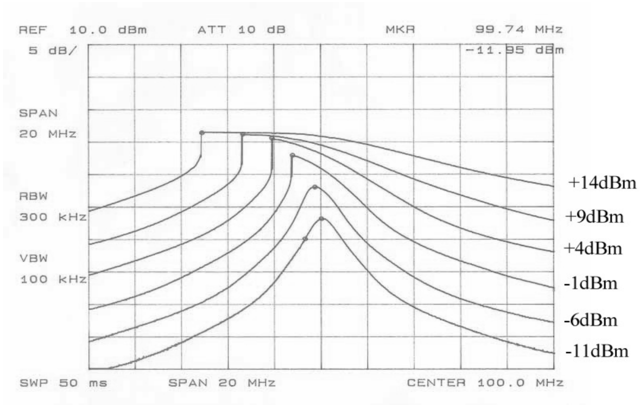

Figure 7. Distortion effects in varactor tuned resonators vs power available at the input QL = 63 Vbias = 3V

IV. Oscillator Resonator Designs

The design techniques have been applied successfully to oscillators using LC (inductor capacitor), transmission line (printed and ceramic), crystal, SAW and Dielectric resonators. Due to length restrictions in this paper a number of resonator configurations will be described and just the essential features will mentioned.

A. LC Resonators

A series and parallel LC resonator configuration will now be described. It is found that due to the typical values of the loss resistance in an LC resonator, very large impedance transformation ratios are required to achieve Q_L/Q_0 ~1/2 for both series and parallel resonators.

One series tuned resonator configuration is shown in Figure 8. This circuit is designed by arranging for the source and load impedances presented to the $L\ R_{LOSS}\ C$ circuit to be $R_{LOSS}/2$ to obtain $Q_L = Q_0/2$ (6dB insertion loss). The inductors merge and the final circuit is shown in Figure 9. Due to the high transformation ratio, The value of the shunt capacitors (C_S) are often high and the effect of the parasitic inductance in the capacitors causes the capacitance to appear even HIGHER. The capacitor values therefore need to be reduced to compensate for this.

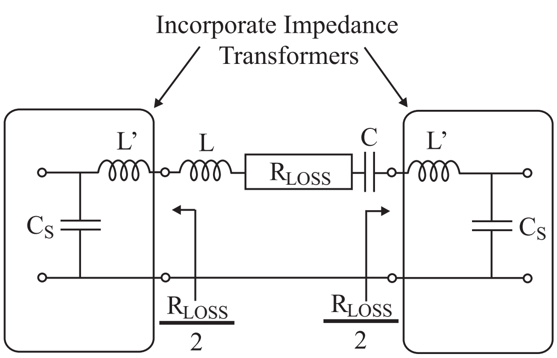

Fig. 8. Resonators with impedance transformers.

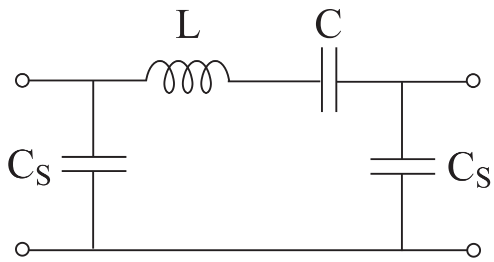

Figure 9, Final Resonator

A parallel tuned circuit is shown in Figure 10. The impedance transformers should present 2R on each side of the resonator. Again parasitic inductance requires a reduction in C_1. Note that this circuit (including the parasitic inductances can also be used to model a CRO design which uses λ/4 short circuit transmission lines [10].

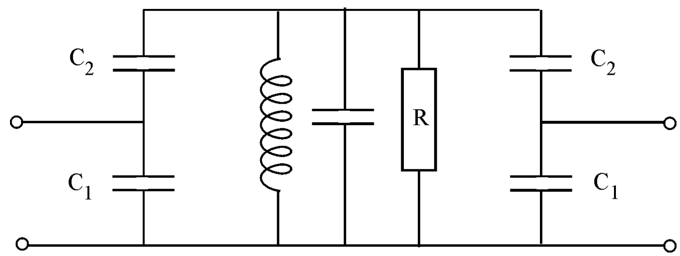

Figure 10, Parallel Tuned resonator (LC and CRO)

B. Crystal and SAW Resonators

Crystal and SAW resonators can be used almost directly (with slight impedance transformations) to obtain the correct Q_L/Q_0. For example the series loss resistance of a crystal is often around 50Ω. Crystal resonators have been used to produce ultra low phase noise performance [11] as illustrated in table 1. Care should be taken in the power handling of these resonators

C. Transmission Line Resonators

Fixed frequency and tunable transmission line resonators have been developed of the form shown in Figure 11. This consists of a ~ λ/2 transmission line with shunt reactances (LL,LC, CC) at either end [12]. This is similar in operation to an optical Fabry Perot Resonator. Design equations for the shunt components in terms of frequency, length, susceptance (B), line loss (α), Q_0 and Q_L/Q_0 have been produced [12].

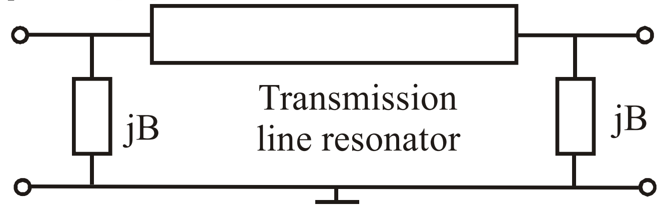

Figure 11 Transmission line resonator equivalent circuit

Printed (Fig 12) [13], helical and printed helical [12] (Fig 13) resonators have been developed some with Qs exceeding 500 at 5GHz and 80 at 21GHz. A printed helical resonator is shown in Figure 12 where each layer has ¾ turn and the via hole is at a standing wave current minimum so the losses are not important. In fact a resistor can be used to suppress even order resonances by up to 30 dB with negligible degradation in the wanted resonance. This was used to produce an all printed oscillator with -120dBc/Hz at 10kHz at 2GHz [12].

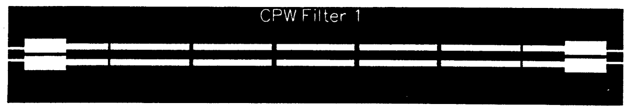

Figure 12 Printed Inductively Coupled Coplanar Transmission Line Filter [13]

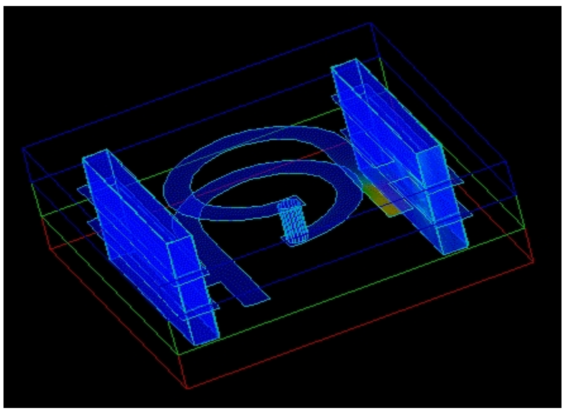

Figure 13 A 3D plot of the shunt inductively coupled resonator [12]

D. *Dielectric Resonators*

Dielectric resonators are typically a cylinder of low loss material with ε_r from 10 (Sapphire and Alumina) to >30 eg $BaTiO_3$. Some of these resonators can be made with zero temperature coefficients. For highest Q these resonators should be kept away from the ground plane [14], [15].

E. *Phase Noise Performance*

The phase noise performance for a number of oscillator types using the theories explained in this paper are shown in Table 1. Significant improvements are expected in future.

Table 1 Phase noise performance of oscillators vs frequency

Freq GHz	Osc Type	Phase Noise 10kHz, dBc/Hz	Noise Floor dBc/Hz	Electronic Tuning	Mod BW
10MHz	Crystal	-123, 1Hz -149, 10Hz	< -160	Few Hz	
1.25	DRO	-173	<-180	Yes	
1.5	CRO	-127	<-165	3MHz	>2MHz
4	DRO	-152	<-170	250kHz	>200kHz
8	DRO compact	-123	<-170	Yes	
10	DRO	-135	<-170	300kHz	>200kHz

V. ACKNOWLEDGEMENTS

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VI. CONCLUSIONS

A simple and accurate linear theory for oscillator design has been described. A set of design rules has been developed and some design examples have been described.

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